

Electroproduction of Transversely Polarized Vector Mesons Via A Quantum Mechanical Anomaly

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By explicit calculation we demonstrate that, in variance with the classical prediction, the leading twist contribution to the exclusive electroproduction of transversely polarized vector mesons from the nucleon does not vanish beyond the leading order in perturbation theory. This appears to be due to a quantum-mechanical anomaly, in the sense that a classical symmetry of the field theory is broken by quantum corrections.

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Recently, there has been considerable interest in the electroproduction of vector mesons from a nucleon by a highly virtual longitudinally-polarized photon [1,9]. Much of this interest follows the study of off-forward parton distributions [10] in the nucleon, which have the potential of providing insights into the nucleon's deep structure. Among others, Collins, Frankfurt and Strikman [4] showed that the reaction amplitude for the diffractive meson electroproduction can be factorized into a form of convolutions of the off-forward parton distributions and meson wave functions with hard scattering coefficients. Hence, it is desirable to study the feasibility to access the off-forward parton distributions in meson electroproduction processes.

Of particular interest is the electroproduction of transversely polarized vector mesons because this may provide a handle to access two twist-2 chiral-odd off-forward parton distribution functions $H_T(x, \xi)$ and $E_T(x, \xi)$. (We refer to Ref. [6] for a complete categorization of the 6 quark and 6 gluon twist-two off-diagonal distribution functions as well as the definitions of $H_T(x, \xi)$ and $E_T(x, \xi)$.) Experimentally, chiral-odd distributions are notoriously difficult to measure because only by matching with some other chiral-odd quantities can they make non-vanishing contributions. As noted in Ref. [4], the leading twist wave function of transversely polarized vector mesons is chirally odd. Thus there arises the possibility of accessing chiral-odd off-forward parton distributions by studying the production of transversely polarized vector mesons.

Unfortunately, it turns out that the hard scattering coefficients associated with matching chiral-odd off-forward parton distributions with chiral-odd vector meson wave functions vanish [7] at leading order in strong coupling (α_s). More interestingly, Diehl, Gousset and

Pire [9] presented a proof that these hard coefficients vanish to *all orders* in perturbation theory. If this is true, it is both good and bad news. The good news is that there will not be any leading twist chiral-odd contaminations in the measurement of chiral-even distributions. The bad news is that at leading twist it is impossible to access the chiral-odd parton distributions by means of vector meson electroproduction.

In this Letter we demonstrate that quantum effects invalidate the proof of vanishing hard coefficients for the transversely polarized vector meson electroproduction. By explicit calculations at one loop level, we show that those hard coefficients do not generally vanish in perturbation theory.

To be specific, consider

$$\gamma^*(q, e_L) + N(P, S) \rightarrow V(K, e_T) + N(P', S'),$$

where the first and second symbols in the parentheses stands for the particle momentum and spin vector, respectively. As usual, we define the average momentum and momentum difference for the initial- and final-state nucleons:

$$\bar{P} = \frac{1}{2}(P + P') , \tag{1}$$

$$\Delta = P' - P . \tag{2}$$

It is most convenient to work in the frame in which \bar{P} and q are collinear with each other and put them in the third direction. Since we are going to deal with the light-cone dominated scattering processes, we introduce two conjugate light-like vectors p^μ and n^μ in the third direction in the sense that $p^2 = n^2 = 0$ and $p \cdot n = 1$. Correspondingly, the relevant momenta can be parameterized as follows:

$$q^\mu = -2\xi p^\mu + \nu n^\mu , \quad (3)$$

$$\bar{P}^\mu = p^\mu + \frac{\bar{M}^2}{2} n^\mu , \quad (4)$$

$$\Delta^\mu = -2\xi(p^\mu - \frac{\bar{M}^2}{2} n^\mu) + \Delta_\perp^\mu , \quad (5)$$

with $\nu = Q^2/(4\xi)$, $Q^2 = -q^2$ and $\bar{M}^2 = M^2 - \Delta^2/4$. In accord with this choice of coordinates, the longitudinal polarization vector of the virtual photon reads

$$e_L^\mu = \frac{1}{Q} (2\xi p^\mu + \nu n^\mu) . \quad (6)$$

At lowest twist we can safely approximate the particle momenta as follows:

$$P^\mu = (1 + \xi)p^\mu + \dots , \quad (7)$$

$$P'^\mu = (1 - \xi)p^\mu + \dots , \quad (8)$$

$$K^\mu = \nu n^\mu + \dots . \quad (9)$$

The basic idea of factorization for vector meson electroproduction is illustrated in Fig. 1. According to the factorization theorem, the dominant mechanism is a single quark scattering process. The reaction amplitude is approximated as a product of three components: the hard partonic scattering, the non-perturbative matrix associated with the nucleon, and the matrix associated with the vector meson production. The active quark has to come back into the nucleon blob after experiencing the hard scattering. On the nucleon side, the initial and final quarks can be thought of carrying momenta $(x + \xi)p$ and $(x - \xi)p$, respectively.

By decomposing the nucleon matrix one has,

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' S' | \bar{\psi}_{\alpha,i} (-\frac{1}{2}\lambda n) \psi_{\beta,j} (\frac{1}{2}\lambda n) | P S \rangle$$

$$= \frac{\delta_{ij}\sigma_{\alpha\beta}^{\rho\lambda}}{4N_c} \left[H_T(x, \xi) \bar{U}(P'S') \sigma_{\rho\lambda} U(PS) + E_T(x, \xi) \bar{U}(P'S') \frac{\gamma_{[\rho} i \Delta_{\lambda]}}{M} U(PS) \right] + \dots \quad (10)$$

To save space, we displayed here only the twist-2 chiral-odd off-forward parton distributions that we will focus on. In the above, α and β are the quark spinor indices, i and j the color indices, $N_c = 3$ is the number of quark colors. By $[\rho\lambda]$ we mean antisymmetrization of the two indices. The ellipses represent all other distribution functions irrelevant for the forthcoming discussion. For simplicity, it has been assumed that there is only one flavor of quark and correspondingly the flavor index is suppressed. Also suppressed is the gauge link operator in the definition of the matrix elements.

On the side of the vector meson production, the momenta that the quark and antiquark carry can be approximately parameterized as $z\nu n$ and $(1-z)\nu n$. Similarly, one can write down the following decomposition for the non-perturbative matrix associated with the vector meson production,

$$\int \frac{d\lambda}{2\pi} e^{-i\lambda z} \langle 0 | \bar{\psi}_{\beta,i}(-\frac{1}{2}\lambda\bar{n}) \psi_{\alpha,j}(\frac{1}{2}\lambda\bar{n}) | K, e_T \rangle = \frac{\delta_{ij}\sigma_{\beta\alpha}^{\rho\lambda}}{2\sqrt{N_c}} F_T(z) K_\rho e_{T\lambda}^* + \dots \quad (11)$$

where $F_T(z)$ is a twist-two chiral-odd vector meson wave function.

The possibility to access chiral-odd quantities arises as $H_T(x, \xi)$ and $E_T(x, \xi)$ are matched with $F_T(z)$ in the closed active quark loop. Without loss of generality, we may put the amplitude into the following form:

$$\begin{aligned} \mathcal{A} = & \left(\frac{e}{Q} \right) \int dx dz \frac{F_T(z)}{(x - \xi + i\epsilon)(1 - z)} \sum_{\text{diagrams}} C_i f_i(x, \xi, z) \\ & \times \left[H_T(x, \xi) \bar{U}(P'S') \not{n} \not{\epsilon}_T U(PS) + E_T(x, \xi) \frac{e_T \cdot \Delta}{M} \bar{U}(P'S') \not{n} U(PS) \right] \quad , \end{aligned} \quad (12)$$

where C_i is the color factor and $f_i(x, \xi, z)$ the corresponding kinematical factor. Our task is to calculate $\sum C_i f_i(x, \xi, z)$ to next-to-leading order in the strong coupling constant.

At the tree level, there are two Feynman diagrams for the hard partonic scattering, as shown in Fig. 2. This corresponds to the fact that either before or after it is struck by the virtual photon, the active quark must undergo a hard scattering to adjust its momentum so as to form the final-state vector meson. (Remember that in our chosen frame, both initial- and final-state nucleons move in the third plus direction, while the vector meson goes in the opposite direction.) By working in the Feynman gauge, one can most easily understand why the hard coefficients $f_i(x, \xi, z)$ vanish at tree level. Actually, it is a direct consequence of the following *4-dimensional* identity:

$$\gamma^\mu \sigma^{\rho\lambda} \gamma_\mu = 0. \quad (13)$$

Here the $\sigma^{\rho\lambda}$ -matrix comes either from the density matrix associated with the off-forward quark helicity-flip distribution in the nucleon (see Fig. 2a) or from that associated with the chiral-odd light-cone wave function of the vector meson (See Fig. 2b), while the two γ -matrices sandwiching $\sigma^{\rho\lambda}$ correspond to the hard gluon scattering exchange. If one sticks to the 4-dimensions, it can be generally shown that the hard coefficients vanish to *all orders* in perturbation theory. We must emphasize, however, that Eq. (13) holds only in the 4-dimensions. If one goes beyond the leading order, loops will necessarily occur and lead to divergences that must be regulated. It is most advantageous to use dimensional regularization for both ultraviolet and infrared divergences. If one works in $(4 - 2\varepsilon)$ dimensions, $\gamma^\mu \sigma^{\rho\lambda} \gamma_\mu$ will be of $\mathcal{O}(\varepsilon)$ and the Dirac trace for the quark loop will no longer be zero. Hence, by canceling the poles from the loop integrations against the ε 's arising from the Dirac traces,

some non-vanishing terms may survive. As will become clear, both ultraviolet and infrared divergences can make contributions to the non-vanishing coefficients at one-loop level.

For convenience, our calculations are done in the Feynman gauge. We found out that it is preferable to group the diagrams by their color structure. We work with renormalized perturbation theory, so all the self-energy and vertex corrections are understood to be accompanied by the corresponding ultraviolet counter-terms.

The diagrams shown in Fig. 3 are characteristic of the three-gluon vertex and possess a common color factor of $C_{\text{fig.3}} = (N_c^2 - 1)/(4\sqrt{N_c})$. The ultraviolet divergences in those vertex corrections are understood to have been canceled by their counter-terms, so only the infrared divergences make contributions. For individual diagrams, there are

$$f_{3a} = -4\alpha_s^2 \left[1 + \frac{1}{2z} \log(1-z) + \frac{\xi}{\xi+x} \log \frac{\xi-x}{2\xi} \right] , \quad (14)$$

$$f_{3b} = \alpha_s^2 \left[-2 - \frac{2}{z} \log(1-z) \right] , \quad (15)$$

$$f_{3c} = \alpha_s^2 \left[-2 - \frac{4\xi}{\xi+x} \log \frac{\xi-x}{2\xi} \right] , \quad (16)$$

$$f_{3d} = -4\alpha_s^2 , \quad (17)$$

$$f_{3e} = -4\alpha_s^2 . \quad (18)$$

Summing over all the five diagrams in Fig. 3, one has

$$\sum_{\text{fig.3}} C_i f_i(\xi, x, z) = -\frac{N_c^2 - 1}{\sqrt{N_c}} \alpha_s^2 \left[4 + \frac{1}{z} \log(1-z) + \frac{2\xi}{\xi+x} \log \frac{\xi-x}{2\xi} \right] . \quad (19)$$

Fig. 4 contains a group of diagrams that have a common color factor but do not contribute to the hard coefficients. The first three drop out simply because their Dirac traces vanish even in the $(4 - 2\varepsilon)$ space. The last two do not contribute because their vertex

corrections contain no infrared divergences, while the ultraviolet divergences are canceled by the counter-terms.

Shown in Fig. 5 are another group of diagrams that have the same color factor $C_{\text{fig. 5}} = -(N_c^2 - 1)/(4N_c^2\sqrt{N_c})$. Some diagrams in this group require lengthy calculation. After considerable algebra, we obtain the contributions for individual diagrams as follows:

$$f_{5a} = \alpha_s^2 \left\{ -\frac{2}{\varepsilon_I} - 4 + 2 \ln \left[\frac{(1-z)(\xi-x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (20)$$

$$f_{5b} = \alpha_s^2 \left\{ -\frac{2}{\varepsilon_I} - 4 + 2 \ln \left[\frac{(1-z)(\xi-x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (21)$$

$$f_{5c} = \alpha_s^2 \left\{ -2 - 2 \frac{1-z}{z} \ln(1-z) \right\}, \quad (22)$$

$$f_{5d} = \alpha_s^2 \left\{ -2 - 2 \frac{\xi-x}{\xi+x} \ln \frac{\xi-x}{2\xi} \right\}, \quad (23)$$

$$f_{5e} = \alpha_s^2 \left\{ -\frac{2}{\varepsilon_I} - \frac{2}{z} \log(1-z) - 2 \log \left[\frac{z^2(\xi-x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (24)$$

$$f_{5f} = \alpha_s^2 \left\{ -\frac{2}{\varepsilon_I} - \frac{4\xi}{\xi+x} \log \frac{\xi-x}{2\xi} - 2 \log \left[\frac{(1-z)(\xi-x)^2}{(2\xi)^2} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (25)$$

$$f_{5g} = \alpha_s^2 \left\{ +\frac{2}{\varepsilon_I} - 2 + 2 \frac{1-z}{z} \log(1-z) + 2 \log \left[\frac{z(\xi+x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (26)$$

$$f_{5h} = \alpha_s^2 \left\{ +\frac{2}{\varepsilon_I} - 2 + 2 \frac{\xi-x}{x+\xi} \log \frac{\xi-x}{2\xi} + 2 \log \left[\frac{z(\xi+x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (27)$$

$$f_{5i} = \alpha_s^2 \left\{ -\frac{4}{\varepsilon_I} - 2 \frac{1-z}{z} \log(1-z) - 2 \frac{\xi-x}{x+\xi} \log \frac{\xi-x}{2\xi} - 4 \log \left[\frac{(1-z)(\xi-x)}{2\xi} \frac{Q^2 e^\gamma}{4\pi\mu^2} \right] \right\}, \quad (28)$$

where $1/\varepsilon_I$ is the infrared pole, μ^2 the scale parameter in the dimensional renormalization, and γ the Euler constant. Summing over all the diagrams in Fig. 5, we have,

$$\sum_{\text{fig.5}} C_i f_i(\xi, x, z) = \frac{N_c^2 - 1}{N_c^2 \sqrt{N_c}} \alpha_s^2 \left[4 + \frac{1}{z} \log(1-z) + \frac{2\xi}{\xi+x} \log \frac{\xi-x}{2\xi} \right]. \quad (29)$$

At this stage, we comment on the one-loop self-energy corrections for the hard scattering partonic processes. Since we work with the renormalized perturbation theory, we need not consider the self-energy insertions either on the incoming or outgoing quark lines. Instead, we need to recalculate the tree diagrams, shown in Fig. 2, in $(4 - 2\varepsilon)$ -dimensions and include a factor of $\sqrt{Z_F}$ for each external quark line of the hard scattering part. The ultraviolet pole in Z_F can be compensated by the ε factor from the tree-level trace in $(4 - 2\varepsilon)$ dimensions. This is exactly where the ultraviolet divergences make their contribution. As a result, we have

$$\sum_{\text{tree}} C_i f_i(\xi, x, z) = \frac{(N_c^2 - 1)^2}{4N_c^2 \sqrt{N_c}} \times 4\alpha_s^2. \quad (30)$$

On the other hand, diagrams with a self-energy insertion onto an internal line do not contribute because they have no infrared divergences.

At this stage, we have exhausted all the one-loop diagrams for the hard scattering process. Combining Eqs. (19), (29) and (30), we finally reach the following compact expression for the one-loop coefficients:

$$\sum_{\text{NTL}} C_i f_i(x, \xi, z) = -4\alpha_s^2 \frac{C_F^2}{\sqrt{N_c}} \left[4 + \frac{1}{z} \log(1 - z) + \frac{2\xi}{\xi + x} \log \frac{\xi - x}{2\xi} \right] \quad (31)$$

with $C_F = (N_c^2 - 1)/(2N_c)$, which is the central result of this Letter.

In summary, our explicit one-loop calculations demonstrate that, within the context of the factorization theorem, the leading twist part of the amplitude for the transversely polarized vector meson electroproduction does not vanish, except at the tree level. The fact that the hard scattering coefficients are finite is directly a consequence of the need to regularize and renormalize perturbation theory. In turn, this is a consequence of the

infinite number of degrees of freedom in a field theory. Therefore, it is a kind of quantum-mechanical anomaly in the sense that the classical symmetry of field theory is broken by quantum corrections.

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Caption

Fig. 1. Illustration of factorization of amplitude for vector meson electroproduction.

Fig. 2. Tree-level hard partonic scattering diagrams.

Fig. 3. One-loop corrections to the hard partonic scattering with a three-gluon vertex.

Fig. 4. A group of one-loop diagrams that share a color factor but do not make non-vanishing contributions to the hard scattering coefficients.

Fig. 5. A group of one-loop diagrams that share a color factor and make non-vanishing contributions to the hard scattering coefficients.















